You are not allowed to use any resources except writing implements to complete this exam. Show your work in the space provided. You have 75 minute.

1. (a) Let $R = \{(1,1), (1,2), (1,3), (2,1), (2,2), (3,1), (3,3)\}$ be a relation on $\{1, 2, 3\}$. Determine whether $R$ is reflexive, symmetric, antisymmetric, or transitive.

   Ans: It’s not antisymmetric since $(1, 2)$ and $(2,1)$ are in $R$, for example. It’s not transitive since $(2,1)$ and $(1,3)$ are in $R$, but $(2,3)$ isn’t. By inspection, $R$ is reflexive on $\{1, 2, 3\}$ and symmetric.

(b) Let $A = \{1, 2, 5, 10\}$. Define $R$ on $A$ by $xRy$ if and only if $y$ is a divisor of $x$. List all the members of $R$, and determine whether $R$ is reflexive, symmetric, antisymmetric, or transitive.

   Ans: $R = \{(1,1), (2,1), (5,1), (10,1), (2,2), (10,2), (5,5), (10,5), (10,10)\}$. $R$ is not symmetric since $(2,1)$ is in $R$, but $(1,2)$ isn’t, for example. By inspection, $R$ is reflexive on $A$, antisymmetric, and transitive (a poset).
2. Prove that $1_A$ is an equivalence relation on $A$.

   Ans: $1_A = \{(a,a): a \text{ in } A\}$ is reflexive on $A$ by definition; i.e., if $x$ is in $A$, then $(x,x)$ is in $1_A$. If $(x,y)$ is in $1_A$, then $y = 1_A(x) = x$, and $(x,y) = (x,x) = (y, x)$; hence, if $(x,y)$ is in $A$, then $(y,x)$ is in $A$; i.e., $1_A$ is symmetric. Similarly, if $(x,y)$ and $(y,z)$ are in $1_A$, then $x = y = z$, and $(x,z)$ is in $1_A$; i.e., $1_A$ is transitive.

3. Prove that $1_A$ is a partial ordering on $A$.

   Ans: We showed in 2, above that $1_A$ is reflexive and transitive. It remains to show that it’s antisymmetric. Suppose $(x,y)$ and $(y,z)$ are in $1_A$. Then $y = 1_A(x) = x$. Hence, $1_A$ is antisymmetric.
4. Let $A = \{1, 2, 3, 6\}$, and define $R$ on $A$ by $xRy$ if and only if $y-x$ is in $\mathbb{N}$.

(a) Is $R$ an equivalence relation on $A$? Justify your answer. If $R$ is an equivalence relation, find $A/R$.

Ans: $R$ is not symmetric since $(1,2)$ is in $R$, but $(2,1)$ isn’t, for example. Thus, $R$ isn’t an equivalence relation.

(b) Is $R$ a partial ordering on $A$? Justify your answer. If $R$ is a partial ordering, draw the Hasse diagram.

Ans: $R = \{(1,1), (1,2), (1,3), (1,6), (2,2), (2,3), (2,6), (3,3), (3,6), (6,6)\}$ is reflexive on $A$, antisymmetric, and transitive, by inspection; hence, $R$ is a partial ordering on $A$. 

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5. Let $A = \{1, 2, 3, 6\}$, and define $R$ on $A$ by $xRy$ if and only if $|y-x| = 3n$ for some $n$ in $\mathbb{N}$.

(a) Is $R$ an equivalence relation on $A$? Justify your answer. If $R$ is an equivalence relation, find $A/R$.

\textbf{Ans:} $R = \{(1,1), (2,2), (3,3), (3,6), (6,3), (6,6)\}$ is reflexive on $A$, symmetric, and transitive, by inspection; hence, $R$ is an equivalence relation on $A$. $A/R = \{\{1\}, \{2\}, \{3,6\}\}$.

(b) Is $R$ a partial ordering on $A$? Justify your answer. If $R$ is a partial ordering, draw the Hasse diagram.

\textbf{Ans:} Since $(3,6)$ and $(6,3)$ are in $R$, $R$ isn’t antisymmetric; hence, $R$ isn’t a partial ordering.