Worksheet 11: Series

Problems

16. Write out the partial sum $S_{N}$ for each series. Do not simplify.
   (a) $\sum_{n=1}^{\infty}(-2/3)^{n}$
   (b) $\sum_{k=1}^{\infty} k^{2}$
   (c) $\sum_{i=1}^{\infty} (3i - 1)$
   (d) $\sum_{j=3}^{\infty} \frac{j}{2}$

17. Identify the value of $p$ and determine whether the series converges for each of the $p$-series, if any, in problem 16. Similarly, determine the ratio $r$ and convergence/divergence of any geometric series.

18. A series starts $1 + \frac{1}{\sqrt{2}} + \cdots$; that is, the first two terms are 1 and $\frac{1}{\sqrt{2}}$. Find a formula for the $n$-th term $a_{n}$ if the series is
   (a) an arithmetic series $\sum_{n=0}^{\infty} (kn + a_{0})$ (i.e., find $k$ and $a_{0}$; $a_{n} = kn + a_{0}$.)
   (b) a geometric series $\sum_{n=0}^{\infty} r^{n}$ (i.e., find $r$; $a_{n} = r^{n}$.)
   (c) a $p$-series $\sum_{n=1}^{\infty} \frac{1}{n^{p}}$ (i.e., find $p$; $a_{n} = \frac{1}{n^{p}}$)

19. Find a formula for the $N$-th partial sum $S_{N}$ of the series $\sum_{k=0}^{\infty} (3k^{2} - k + 1)$, and show that the series diverges.

20. Give two different decimal digit sequences, $(d_{n})$ and $(D_{n})$, that give decimal representations, $\sum_{n=1}^{\infty} d_{n}10^{-n}$ and $\sum_{n=1}^{\infty} D_{n}10^{-n}$, for 3/4.

21. Find a formula for the $N$-th partial sum of $\sum_{n=0}^{\infty} 5^{n}$, and show that the series diverges.

22. Determine whether the series converges or diverges. Evaluate, if the series converges.
   (a) $\sum_{n=1}^{\infty} \frac{1}{n} (-2/3)^{n}$
   (b) $\sum_{n=1}^{\infty} \frac{5^{n}}{2^{n}}$
   (c) $\sum_{n=1}^{\infty} \frac{1}{n^{2}}$
   (d) $\sum_{n=1}^{\infty} \frac{1}{(1/3)^{n}}$

23. Determine whether the series converges or diverges. $\gamma$ is Euler's constant. $\gamma \approx 0.577216$.
   (a) $\sum_{n=1}^{\infty} \frac{1}{n^{\gamma}}$
   (b) $\sum_{n=1}^{\infty} n^{\gamma}$
   (c) $\sum_{n=1}^{\infty} n^{1/\gamma}$
   (d) $\sum_{n=1}^{\infty} \frac{1}{n^{\gamma}}$

24. Determine whether the series converges or diverges. Evaluate, if the series converges.
   (a) $\sum_{n=1}^{\infty} \left( \frac{1}{n^{\gamma}} - \frac{1}{(n+1)^{\gamma}} \right)$
   (b) $\sum_{n=1}^{\infty} \frac{1}{2n}$
   (c) $\sum_{n=1}^{\infty} (-1)^{n} \frac{1}{n}$
   (d) $\sum_{n=2}^{\infty} (-1)^{n+1} \frac{1}{n}$

25. Find a geometric series that converges to 3/4.

26. Is there a geometric series that converges to 1/3? If there is, give it. If there isn't, determine all the real numbers that are not given by a geometric series.

27. Show that the telescoping series $\sum_{n=0}^{\infty} \lceil n! - (n + 1)! \rceil$ diverges.

28. Find a formula for the coefficients $a_{n}$ in the power series representation $\sum_{n=0}^{\infty} a_{n}x^{n}$ for $e^{2x}$. (Recall that $e^{x} = \sum_{n=0}^{\infty} \frac{1}{n!} x^{n}$)

29. Find the Fourier series for $f(x) = 2x$. (WS#10, Problem 15 might be useful here.)

30. Find the Fourier Series for $f(x) = 2x$. (WS#10, Problem 15 might be useful here.)

Selected Answers to Problems

To be announced after the homework is collected.

17. $\sum_{k=1}^{\infty} k^{2}$ is a $p$-series with $p = 2$; it diverges since $p \leq 1$. $\sum_{n=0}^{\infty} (-2/3)^{n}$ is a geometric series with $r = -2/3$; it converges since $-1 < r < 1$.

18. (a) $\sum_{n=0}^{\infty} ((\frac{1}{\sqrt{2}} - 1)n + 1)$
   (b) $\sum_{n=0}^{\infty} (\frac{1}{\sqrt{2}})^{n}$
   (c) $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$

19. $S_{N} = \frac{3N(N+1)(2N+1)}{6} - \frac{N(N+1)}{2} + (N + 1) \to \infty$ as $N \to \infty$.

20. (a) converges to 3/5
   (b) converges to -3
   (c) converges to -1
   (d) diverges

21. $S_{N} = \frac{1-5^{N+1}}{1-5} \to \infty$ as $N \to \infty$.

22. (a) converges to 3/5
   (b) converges to -3
   (c) converges to -1
   (d) diverges

23. (d) is the only one that converges

24. (a) converges to 1
   (b) diverges
   (c) converges to $-\ln 2$
   (d) converges to $\ln 2 - 1$

25. $\sum_{n=0}^{\infty} (-1/3)^{n}$

26. There is no geometric series that converges to 1/3. The values that aren't given by geometric series are $(-\infty, 1/2]$.

27. $S_{N} = 1 - (N + 1)! \to -\infty$

29. $3 + \sin 2x - 5 \cos 8x$ is its own Fourier series.