Worksheet 15: Calculus of Power Series

Problems
11. Give the power series centered at 0 and its interval of convergence for each of the following functions. Memorize these series. Finally, use the power series to write the given number as a series.

(a) \( \frac{1}{1-x} \); 2.
(b) \( \ln(1 + x) \); \( \ln(1/2) \).
(c) \( e^x \); \( \sqrt{e} \).
(d) \( \sin x \); 1/2.
(e) \( \cos x \); 1/2.

Use known power series (Problem 11) to find the power series centered at 0 for each of the functions in Problems 12-15. Find the interval of convergence in each case.

12. \( \frac{1}{1-3x} \)
13. \( \frac{1}{1-x} \)
14. \( \sinh x \)
15. \( 2x \cos(x^2) \)

Identify the functions given by the power series in Problems 16-19.

16. \( 1 + x^2 + x^4 + x^6 + \cdots \)
17. \( \frac{x^2}{3!} + \frac{x^4}{5!} + \frac{x^6}{7!} + \cdots \)
18. \( \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots \)
19. \( -1 + x - \frac{(x-1)^3}{3!} + \frac{(x-1)^5}{5!} - \frac{(x-1)^7}{7!} + \cdots \)

20. Let \( f(x) = \sum_{n=0}^{\infty} f_n x^n \), where \( (f_n) \) is the Fibonacci sequence. Find the power series centered at 0 for \( f'(x) \), and determine its domain.