Problems
16. Write out the partial sum $S_n$ for each series. Do not simplify.
   (a) $\sum_{n=0}^{\infty} (-2/3)^n$
   (b) $\sum_{k=1}^{\infty} k^2$
   (c) $\sum_{i=1}^{\infty} (3i - 1)$
   (d) $\sum_{j=3}^{\infty} \frac{j}{2^j}$

17. Identify the value of $p$ and determine whether the series converges for each of the $p$-series, if any, in problem 16. Similarly, determine the ratio $r$ and convergence/divergence of any geometric series.

18. A series starts $1 + \frac{1}{\sqrt{2}} + \cdots$; that is, the first two terms are $1$ and $\frac{1}{\sqrt{2}}$. Find a formula for the $n$-th term $a_n$ if the series is
   (a) an arithmetic series $\sum_{n=0}^{\infty} (kn + a_0)$ (i.e., find $k$ and $a_0$; $a_n = kn + a_0$.)
   (b) a geometric series $\sum_{n=0}^{\infty} r^n$ (i.e., find $r$; $a_n = r^n$.)
   (c) a $p$-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ (i.e., find $p$; $a_n = \frac{1}{n^p}$.)

19. Find a formula for the $N$-th partial sum $S_N$ of the series $\sum_{k=0}^{\infty} (3k^2 - k + 1)$, and show that the series diverges.

20. Give two different decimal digit sequences, $(d_n)$ and $(D_n)$, that give decimal representations, $\sum_{n=1}^{\infty} d_n 10^{-n}$ and $\sum_{n=1}^{\infty} D_n 10^{-n}$, for $3/4$.

21. Find a formula for the $N$-th partial sum of $\sum_{n=0}^{\infty} 5^n$, and show that the series diverges.

22. Determine whether the series converges or diverges. Evaluate, if the series converges.
   (a) $\sum_{n=1}^{\infty} \frac{1}{n^2}$
   (b) $\sum_{n=1}^{\infty} \frac{1}{n^3}$
   (c) $\sum_{n=1}^{\infty} \frac{1}{n^4}$
   (d) $\sum_{n=0}^{\infty} \frac{1}{(n/5)^n}$

23. Determine whether the series converges or diverges. $\gamma$ is Euler’s constant. $\gamma \approx 0.577216$.
   (a) $\sum_{n=1}^{\infty} \frac{1}{n^\gamma}$
   (b) $\sum_{n=1}^{\infty} n^\gamma$
   (c) $\sum_{n=1}^{\infty} n^{1/\gamma}$
   (d) $\sum_{n=1}^{\infty} \frac{1}{n^{1/\gamma}}$

24. Determine whether the series converges or diverges. Evaluate, if the series converges.
   (a) $\sum_{n=1}^{\infty} \left( \frac{1}{n} - \frac{1}{(n+1)^2} \right)$
   (b) $\sum_{n=1}^{\infty} \frac{1}{2^n}$
   (c) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$
   (d) $\sum_{n=2}^{\infty} (-1)^n + 1 \frac{1}{n}$

25. Find a geometric series that converges to $3/4$.

26. Is there a geometric series that converges to $1/3$? If there is, give it. If there isn’t, determine all the real numbers that are not given by a geometric series.

27. Show that the telescoping series $\sum_{n=0}^{\infty} [n! - (n + 1)!]$ diverges.

28. Find a formula for the coefficients $a_n$ in the power series representation $\sum_{n=0}^{\infty} a_n x^n$ for $e^x$. (Recall that $e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n$.)

29. Find the Fourier series for $f(x) = 2x$. (WS#10, Problem 15 might be useful here.)

Selected Answers to Problems To be announced after the homework is collected.

17. $\sum_{k=1}^{\infty} k^2$ is a $p$-series with $p = 2$; it diverges since $p \leq 1$. $\sum_{n=0}^{\infty} (-2/3)^n$ is a geometric series with $r = -2/3$; it converges since $-1 < r < 1$.

18. (a) $\sum_{n=0}^{\infty} ((\frac{1}{\sqrt{2}} - 1)n + 1)$ (b) $\sum_{n=0}^{\infty} \left( \frac{1}{\sqrt{2}} \right)^n$

19. $S_N = 3N(N+1)(2N+1) - N(N+1) + N \to \infty$ as $N \to \infty$.

20. (a) converges to $3/5$ (b) converges to $-3$ (c) converges to $-1$ (d) diverges

21. $S_N = 1 - (N+1)! \to -\infty$.

22. (a) converges to $3/5$ (b) converges to $-3$ (c) converges to $-1$ (d) diverges

23. (d) is the only one that converges

24. (a) converges to $1$ (b) diverges (c) converges to $-\ln 2$ (d) converges to $\ln 2 - 1$

25. $\sum_{n=0}^{\infty} (1/3)^n$

26. There is no geometric series that converges to $1/3$.

The values that aren’t given by geometric series are $(-\infty, 1/2]$.

27. $S_N = 1 - (N+1)! \to -\infty$

28. $3 + \sin 2x - 5 \cos 8x$ is its own Fourier series.

30. $\sum_{n=0}^{\infty} (-1)^n + 1 \frac{1}{n} \sin nx$