**Worksheet 11: Series**

**Problems**

16. Write out the partial sum $S_4$ for each series. Do not simplify.
   (a) $\sum_{n=0}^{\infty} (-1/2)^n$  
   (b) $\sum_{k=0}^{\infty} 2^k$  
   (c) $\sum_{i=1}^{\infty} (3 - 5i)$  
   (d) $\sum_{j=1}^{\infty} (2j)!/j^2$  
   (e) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

17. Identify the value of $p$ and determine whether the series converges for each of the $p$-series, if any, in problem 16. Similarly, determine the ratio $r$ and convergence/divergence of any geometric series.

18. A series starts $1 + \frac{1}{2} + \cdots$; that is, the first two terms are 1 and $\frac{1}{2}$. Find a formula for the $n$-th term $a_n$ if the series is
   (a) an arithmetic series $\sum_{n=0}^{\infty} (kn + a_0)$ (i.e., find $k$ and $a_0$; $a_n = kn + a_0$).  
   (b) a geometric series $\sum_{n=0}^{\infty} r^n$ (i.e., find $r$; $a_n = r^n$).  
   (c) a $p$-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ (i.e., find $p$; $a_n = \frac{1}{n^p}$).

19. Find a formula for the $N$-th partial sum of $\sum_{n=0}^{\infty} (3/2)^n$. Does the series converge or diverges? Explain.

20. Determine whether the series converges or diverges. Evaluate, if the series converges.
   (a) $\sum_{n=0}^{\infty} \frac{3^n}{5^n}$  
   (b) $\sum_{n=0}^{\infty} (-3/5)^n$  
   (c) $\sum_{n=1}^{\infty} \frac{3^n}{5^n}$  
   (d) $\sum_{n=0}^{\infty} (1/7)^{-n}$

21. Determine whether the series converges or diverges. $e$ is the Euler number. $e \approx 2.7$.
   (a) $\sum_{n=1}^{\infty} \frac{1}{n^n}$  
   (b) $\sum_{n=1}^{\infty} n^e$  
   (c) $\sum_{n=1}^{\infty} n^{1/e}$  
   (d) $\sum_{n=1}^{\infty} n^{-1/e}$

22. Determine whether the series converges or diverges. Evaluate, if the series converges.
   (a) $\sum_{n=1}^{\infty} \left( \frac{2}{n^n} - \frac{2}{(n+1)^n} \right)$  
   (b) $\sum_{n=1}^{\infty} \frac{3^n}{5^n}$  
   (c) $\sum_{n=3}^{\infty} (-1)^{n+1} \frac{1}{n}$  
   (d) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n+1}$

23. Find a geometric series that converges to $3/4$. Is there a geometric series that converges to $1/3$? If there is, give it. If there isn’t, determine all the real numbers that are given by a geometric series.

24. Find a formula for the $N$-th partial sum of the telescoping series $\sum_{n=0}^{\infty} [n!2^n - (n+1)!2^{n+1}]$, and use the formula and a calculator to find the partial sums $S_0$, $S_1$, $S_2$, $S_3$. What appears to be the value of the series?

25. Find a formula for the coefficients $a_n$ in the power series representation $\sum_{n=0}^{\infty} a_n x^n$ for $e^{-x}$. (Recall that $e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n$.

**Answers to Selected Problems**

17.(e) is a $p$-series with $p = 1/2$, hence it diverges. Geometric series are (a) $r = -1/2$, convergent, and (b) $r = 2$, divergent.

18. (a) $\sum_{n=0}^{\infty} ((-3/4)n + 1)$  
   (b) $\sum_{n=0}^{\infty} (1/4)^n$  
   (c) $\sum_{n=1}^{\infty} \frac{1}{n}$

19. $S_1 = (1 - (3/2)^{n+1})/(1 - 3/2)$ diverges since $(3/2)^{n+1} \to \infty$.

20. (a) Converges to $-15/4$. (b) Converges to $5/8$. (c) Converges to $-15/4 - 3 = -3/4$ (d) Diverges.

21. (a) is the only convergent one of the four.

22. (a) Converges to $2$. (b) Diverges. (c) Converges to $\ln 2 - 1/2$ (d) Converges to $\ln 2 - 1$

23. The geometric series with ratio $r = -1/3$ converges to $3/4$. The possible sums of the geometric series are the numbers in the interval $(1/2, \infty)$.

24. $S_N = 1 - (N+1)!2^{N+1}$, $S_0 = -1$, $S_1 = -7$, $S_2 = -47$, $S_3 = -383$, and $S_N \to -\infty$.

25. $e^{-x} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^n$