Applications of the Integral
1. Find the length of the curve \( y = \sqrt{x^3} \) from \( x = 0 \) to \( x = 3 \).
2. Find the length of the parametric curve \( x = \cos(e^{2t}), \ y = \sin(e^{2t}) \) for \( 0 \leq t \leq 1 \).
3. Write an integral for the area between the curves \( y = \sinh x \) and \( y = 1 - x^2 \) for \( -3 \leq x \leq 12 \). You need not evaluate the integral.
4. Find the area between the curves \( y = x^2 + 1 \) and \( y = 3 - x \) from \( x = -4 \) to \( x = 4 \).
5. Use an integral to show that the surface area of a sphere of radius \( r \) is \( 4\pi r^2 \).
6. Consider the bounded region \( \mathcal{R} \) in the plane bordered by the curves \( y = e^x, \ y = 1, \) and \( x = 1 \). Find the volume of the solid whose base is \( \mathcal{R} \) and whose cross-sections perpendicular to the \( x \)-axis are squares.
7. Write and evaluate an integral for the volume of the solid generated by revolving \( \mathcal{R} \) in problem 6
   (a) about the \( x \)-axis.  
   (b) about the \( y \)-axis.

Sequences
8. For the sequence \( (5n - 1)_{n=-3}^{\infty} \), list the first 5 terms and find the (-1)-th, 0-th, 3-th, and 52-th term.
9. Give a formula for the \( n \)-th term of a sequence whose first 5 terms are 2,3,5, 7 and 11.
    Give the next 3 terms. According to your formula, find the 100-th term.
10. Find the 7-th term \( g_7 \) in the golden ratio sequence and the 10-th term \( f_{10} \) in the Fibonacci sequence.
11. For a non-negative integer \( n \) and real number \( x \), Pochhammer’s Symbol \( (x)_n \) is defined recursively by
    \[
    (x)_0 = 1, \quad \text{and} \quad (x)_n = (x + n - 1) \times (x)_{n-1}, \quad \text{for} \quad n = 1, 2, 3, \ldots .
    \]
    Consider the sequence with \( n \)-th term \( a_n = (-4)_n \), for \( n = 1, 2, 3, \ldots \). List the first 7 terms of the sequence, and find \( a_{271} \).
12. Find the limit, if the sequence converges. Cite the basic sequence limit rule, or rules, from Part A.3 of Worksheet 10 you used.
    (a) \( \left( \frac{4 - 2n^{1/n}}{3 + (1/2)^n} \right)_{n=1}^{\infty} \)
    (b) \( \left( 6 - \frac{e^n}{f_n} \right) \), where \( f_n \) is the \( n \)-th term of the Fibonacci sequence.
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(c) \( \left( \frac{-2g^n}{(\pi + 1)^n} \right) \), where \( g \) is the golden ratio.

(d) \( 3g_n - \frac{2}{g_{n+2}} \), where \( (g_n) \) is the golden ratio sequence.

(e) \( \left( 1 + \frac{3}{n} \right)^{-4n} \)

13. The \( n \)-th term of a sequence that involves a rational function is given. Find the limit, if the sequence converges.

(a) \( \frac{2n^3 - n^2}{3n - n^3} \)

(b) \( \frac{6 - n^2}{n^3 - 10} \)

(c) \( \frac{4n^3 - 2n + 1}{5n^2 + 7n - 3} \)

(d) \( \sqrt{\frac{4n^2 + 1}{64n^2 + 3n + 1}} \)

(e) \( \ln(n^4 - 1) - 4 \ln(2n + 1) \)

14. Find a formula for the \( n \)-th moment \( \mu_n \) corresponding to the weight function \( w(x) = x^2 \) on the interval \([-1, 1]\).

15. Find a formula for the Fourier coefficients \( a_n \) and \( b_n \) for the function \( f(x) = 5x^2 \).

Series

16. Write out the partial sum \( S_5 \) for each series. Do not simplify.

(a) \( \sum_{n=0}^{\infty} (-1/4)^n \)  (b) \( \sum_{k=1}^{\infty} k^{-3} \)  (c) \( \sum_{i=2}^{\infty} (2 - 3i) \)  (d) \( \sum_{j=0}^{\infty} \frac{2^j}{j!} \)  (e) \( \sum_{m=1}^{\infty} ((1/2)^m - (1/2)^{m-1}) \)

17. A series starts \( 1 + \sqrt{2} + \cdots \); that is, the first two terms are 1 and \( \sqrt{2} \). Find a formula for the \( n \)-th term \( a_n \), if the series is

(a) an arithmetic series \( \sum_{n=0}^{\infty} (dn + a_0) \) (i.e., find \( d \) and \( a_0 \); report \( a_n = dn + a_0 \)).

(b) a geometric series \( \sum_{n=0}^{\infty} r^n \) (i.e., find \( r \); report \( a_n = r^n \)).

(c) a \( p \)-series \( \sum_{n=1}^{\infty} \frac{1}{n^p} \) (i.e., find \( p \); report \( a_n = \frac{1}{n^p} \)).

18. Find a formula for the \( N \)-th partial sum \( S_N \) of the series \( \sum_{k=0}^{\infty} (2k^2 - 3k + 1) \), and show that the series diverges.

19. (a) Find a formula for the \( N \)-th partial sum of \( \sum_{n=0}^{\infty} (1/3)^n \), show that the series converges, and find its value. (b) Is there a geometric series that converges to \( 3/5 \)? If there is, give the series. If there isn’t, determine all the real numbers that are not given by a geometric series.

20. Show that the telescoping series \( \sum_{m=1}^{\infty} ((1/2)^m - (1/2)^{m-1}) \) converges and find its value.
Selected Answers

1. \( \frac{8}{27} (\sqrt{(31/4)^3} - 1) \) 
2. \( e^2 - 1 \) 
4. \( 107/3 \) 
6. \( \frac{e^2}{2} - 2e + \frac{5}{2} \) 
7. (a) \( \frac{\pi}{2} (e^2 - 3) \) (b) \( \pi \)

8. \( (5n - 1)_{n=-3}^{\infty} = (-16, -11, -6, -1, 4, \ldots) \) 
9. The cheating bastard way is \( a_1 = 2, a_2 = 3, a_3 = 5, a_4 = 7, a_5 = 11, \) and \( a_n = 0 \) for \( n = 6, 7, 8, \ldots \). Then \( a_{100} = 0. \)

10. \( f_{10} = 89, g_7 = 34/21 \) 
11. \( ((-4)_n)_{n=-3}^{\infty} = (-4, 12, -24, 24, 0, 0, 0, \ldots) \)
12. (a) \( 2/3 \) by rules (d) and (e) (b) diverges to \( -\infty \) (c) 0 by rule (e) with \( r = 2g/(\pi + 1) \) (d) \( 3g - (2/g) \) by rule (b) (e) \( e^{-12} \) using rule (a) and a substitution.

13. (a) -2 (b) 0 (c) diverges to \( \infty \) (d) \( 1/4 \) (e) \( \ln(1/16) \) 
14. 0 if \( n \) is odd, and \( \frac{2}{n+3} \) if \( n \) is even.
15. \( b_n = 0 \) for all \( n, a_0 = \frac{10}{3} \pi^2, \) and \( a_n = (-1)^n \frac{20}{n^2} \) for \( n = 1, 2, 3, \ldots \).