FINAL EXAM

Follow all instructions and show your work in the space provided. There are 10 problems that will be scored from a total of 200 points. You are not allowed to use any resource except writing implements. You have 2 hours to complete the exam.

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1. Give a formula \( f(x) \) for a function with the following features:
   \( f(x) \) is a rational function
   \( f(x) \) is an odd function
   \( f(x) \) has vertical asymptotes at \( x = \pm 5 \)
   \( f(x) \) has no horizontal asymptotes
   \( f(x) \) has value \(-3\) at \( x = 1 \)

   A possible solution:
   \[
   f(x) = \frac{25}{x + 5} - \frac{7}{x - 5} \]

2. \( f(x) = \begin{cases} 
0, & \text{if } x \leq 2 \\
mx + b, & \text{if } 2 \leq x \leq 3 \\
2, & \text{if } 3 \leq x 
\end{cases} \)

   Find \( m \) and \( b \) so that \( f(x) \) is continuous for all \( x \).

3. \( f(x) = \begin{cases} 
x^3, & \text{if } x \leq 1 \\
x^2 + b, & \text{if } 1 < x < 3 \\
x, & \text{if } 3 \leq x 
\end{cases} \)

   Find \( m \) and \( b \) so that \( f(x) \) is differentiable for all \( x \).

4. Calculate \( y' \).
   (a) \( y = 2 - x^2 + \sec x + 7 \ln x \)
   (b) \( y = e^{-\cos 2x} \)
   (c) \( y = x/\sin x \)
   (d) \( y = xe^x \tan x \)

5. Find two positive numbers, \( x \) and \( y \), such that their product is 1, and \( 5x + y \) is a minimum.
6. Evaluate the indefinite integrals.

(a) \[ \int 2 \sin t \cos t \, dt \]

(b) \[ \int (7x^{-1/2} + 3x^{-1}) \, dx \]

(c) \[ \int e^{\ln r} \, dr \]

(d) \[ \int \frac{(w^3 - 1) \, dw}{w^2} \]

(e) \[ \int \frac{3u^2 + 8u - 16}{u + 4} \, du \]

7. \( f(x) \) is an even function with domain all \( x \)'s, \( \int_{-6}^{6} f(x) \, dx = -5 \) and \( \int_{-5}^{5} f(x) \, dx = 11 \). Evaluate the following.

(a) \[ \int_{-6}^{6} f(x) \, dx \]

(b) \[ \int_{-6}^{6} (4f(x) - 3x) \, dx \]

(c) \[ \int_{0}^{5} f(x) \, dx \]

(d) \[ \int_{-5}^{5} f(x) \, dx \]

(e) \[ \int_{-5}^{5} f(x) \, dx \]

8. Find \( \lim_{n \to \infty} \sum_{k=1}^{n} \left( \frac{3k}{n} \right)^2 \left( \frac{3}{n} \right) \) by writing it as an integral, then evaluating using the FTC.
9. Solve the following.

(a) \( \sum_{i=1}^{15} a_i = 150 \). Evaluate \( \sum_{i=1}^{15} (3a_i - 1) \).

(b) Evaluate \( \sum_{j=1}^{3} \cos(\pi / j) \).

(c) Evaluate \( \sum_{n=1}^{100} n \).

(d) Write \( \int_{0}^{3} x^2 \, dx \) as a limit of a Riemann sum, using regular partitions and right endpoints.

(e) Write \( \frac{2}{3} - \frac{3}{4} + \frac{1}{2} - \frac{5}{6} + \frac{6}{7} \) in sigma notation.

10. Evaluate and simplify the definite integrals.

(a) \( \int_{-\pi/6}^{\pi/2} \cos t \, dt \)

(b) \( \int_{-5\pi}^{5\pi} x^2 \sin x \, dx \)

(c) \( \int_{0}^{\ln 3} e^w \, dw \)

(d) \( \int_{0}^{\tan(\frac{x}{3})} dx \)

(e) \( \int_{-4}^{4} \sqrt{16 - x^2} \, dx \)