EXAM 2 (Solved)

Follow all instructions and show your work in the space provided. There are five problems in all, for a total of 100 points. Do not write in the table on the right. It will be used to tabulate your score. You will be given 10 minutes at the beginning of the exam period to use your calculator on Problem 1. Complete the rest of the exam without a calculator. You have 60 minutes to complete the entire exam.

1. Use your calculator to find approximate values for the following.
   (a) \( 1 - \frac{\pi}{2} \approx -0.570796327 \)
   (b) \( \tan \frac{\pi}{3} = 0.93159646 \)
   (c) \( \cot 0.01^\circ \approx 5729.577893 \)
   (d) \( \sec(-\frac{\pi}{4}) \approx 1.411094471 \)
   (e) \( \csc \frac{\pi}{4} \approx 72.9535369 \)

2. A right triangle has a hypotenuse that’s 4 times the length of the base \( b \).
   (a) Make a labeled drawing of the triangle and find the area \( A \) as a function of \( b \) only.

   \[
   \text{The height is } \sqrt{(4b)^2 - b^2} = \sqrt{15}b \text{, using the Pythagorean Theorem.}
   \]

   \[
   \text{The area is } A = \frac{1}{2}(base)(height) = \frac{1}{2}b\sqrt{15}b = \frac{\sqrt{15}b^2}{2}.
   \]

   (b) Find the values of each of the 6 standard trigonometric functions of the angle \( \theta \) made by the base and the hypotenuse of the triangle in part a.

   \[
   \sin \theta = \sqrt{15}/4 \quad \quad \quad \csc \theta = 4/\sqrt{15}
   \]

   \[
   \cos \theta = 1/4 \quad \quad \quad \quad \quad \sec \theta = 4
   \]

   \[
   \tan \theta = \sqrt{15} \quad \quad \quad \cot \theta = 1/\sqrt{15}
   \]
3. Find the derivatives.

(a) \[ \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}} \]

(b) \[ \frac{d}{dx} \left( \frac{1}{\sqrt[3]{x^4}} \right) = -\frac{4}{3} x^{-7/3} = -\frac{4}{3\sqrt[3]{x^7}} \]

(c) \[ \frac{d}{dx} \left( \frac{\cos x}{x} \right) = \frac{(x)(-\sin x) - (\cos x)(1)}{x^2} \]

(d) \[ \frac{d}{dx} \left( \ln(x^4 + 1) \right) = \frac{1}{x^4 + 1} \cdot 4x^3 \]

(e) \[ \frac{d}{dx} (\log x) = \frac{1}{x \ln 10} \]

(f) \[ \frac{d}{dx}(2^x) = 2^x \ln 2 \]

(g) \[ \frac{d}{dx}(xe^x) = (1)e^x + x(e^x) = xe^x + xe^x \]

(h) \[ \frac{d}{dx}(\pi^x) = 0 \]

(i) \[ \frac{d}{dx}(\tan^2 x) = 2 \tan x \sec^2 x \]

(j) \[ \frac{d}{dx} \left( \frac{1}{x^5} \right) = -\frac{5}{x^6} \]

Solve the following.

(k) What is the Holç Rule, and why did we name it that?

The Holç Rule is the derivative rule \[ \frac{d}{dx}(\ln x) = \frac{1}{x} \], and it got its name because

the Power Rule \[ \frac{d}{dx}(x^r) = rx^{r-1} \] produces ever other power of \( x \) except

\[ x^{-1} = \frac{1}{x} \], hence the Holç Rule fill that hole.

(l) For \( y = x \ln x \), find \( y^{\prime\prime}(x) \).

\[ y' = (1)\ln x + x\left(\frac{1}{x}\right) = \ln x + 1 \], so \( y'' = \frac{1}{x} \), and hence \( y''' = \frac{-1}{x^2} \).

(m) For \( f(x) = x^{101} + x^{100} - e^x + \sin x \), find \( f^{(101)}(x) \).

\[ f^{(101)}(x) = 101! - e^x + \cos x \]
4. (a) List 5 features of a function given by a graph that might indicate that the function fails to have a derivative at a given point.

(i) vertical tangent
(ii) jump
(iii) vertical asymptote
(iv) hole
(v) point jump (there are many others)

(b) Describe the “Happy Land of Functions” as discussed in class to this point in our studies. A picture with representative functions will suffice.

\[ H(x) \quad \text{(Heaviside function)} \]

5. Find the approximate value of \( \tan(3/4) \) given by the local linearization of \( f(x) = \tan x \) near \( x = \pi/4 \). (Hint: See Problem 1.)

\[ m = f'(\frac{\pi}{4}) = \sec^2 \frac{\pi}{4} = 2, \] and the point of tangency is \( (\frac{\pi}{4}, f(\frac{\pi}{4})) = (\frac{\pi}{4}, 1) \). Thus, the equation of the tangent is \( y - 1 = 2(x - \frac{\pi}{4}) \) or \( y = 2x + 1 - \frac{\pi}{2} \). By Problem 1, part a, \( 1 - \frac{\pi}{2} \approx -0.57 \). Therefore, \( \tan x \approx 2x - 0.57 \) near \( x = \frac{\pi}{4} \). In particular,

\[ \tan(3/4) \approx 2(3/4) - 0.57 = 1.5 - 0.57 = 0.93. \]