Notes About Exponential Functions:

<table>
<thead>
<tr>
<th>item</th>
<th>explanation</th>
</tr>
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<tbody>
<tr>
<td>general form</td>
<td>( y = y_0 b^x ) ( (y_0 \neq 0, ; 0 &lt; b &lt; 1 ; \text{or} ; 1 &lt; b) )</td>
</tr>
<tr>
<td>initial value</td>
<td>( y_0 = y(0), \text{the value of } y \text{ when } x = 0 )</td>
</tr>
<tr>
<td>growth factor</td>
<td>( b ), the base of the exponential</td>
</tr>
<tr>
<td>domain</td>
<td>all real ( x )</td>
</tr>
<tr>
<td>range</td>
<td>if ( y_0 &gt; 0 ), then range is ( y &gt; 0 )</td>
</tr>
<tr>
<td>asymptotes</td>
<td>( y = 0 ) ( (x \text{-axis}) )</td>
</tr>
<tr>
<td>graph</td>
<td>for ( 0 &lt; y_0 ) and ( 1 &lt; b ) . . .</td>
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defining qualities

(1) Geometrically . . .

for \( 0 < y_0 \) and \( 1 < b \) . . .

area under the curve from \(-\infty\) to \( x \) is proportional to \( y(x) \)

(2) Mathematically . . .

for \( y_0 = 1 \), so that \( y(x) = b^x \) . . .

\( y(x_1 + x_2) = y(x_1) \cdot y(x_2) \)

Important Example:
The natural exponential function \( y = e^x \) (best studied with series)
Euler Number \( e \) \( (e \text{ is irrational}; \; e \approx 2.71828182) \)
model applications: exponential growth and decay, continuously compounded interest
Exercises.

1. Rewrite each of the following in the form $y_a b^x$, for constants $y_a$ and $b$.

(a) $\frac{7^x}{5}$  
(b) $\frac{-3}{4^x}$  
(c) $4^{-5x}$

(d) $3^{-x}$  
(e) $\frac{2^x}{3^x}$  
(f) $-6^{2x}$

(g) $2^{x+1}$  
(h) $7^{3x-2}$  
(i) $-4^{2-x}$

2. Which of the following are exponential functions?

(a) $y = 5 \cdot 7^x$  
(b) $f(x) = -2^{-3x}$  
(c) $g(x) = 3\sqrt{x}$  
(d) $y(x) = -(2x)^3$

(e) $e(x) = -5^{3x-1}$  
(f) $f(x) = 5^{7x^2-100}$  
(g) $y = 4^x + x^2$  
(h) $y = 4^x \cdot x^2$

(i) $y = x^4$  
(j) $f(x) = 7x^2 - 100$  
(k) $y = 2^x / 3^x$  
(l) $y = 4^x / x^2$

3. Determine the initial value, growth factor, and range for each of the exponential functions in exercise 2.

<table>
<thead>
<tr>
<th>exponential function</th>
<th>initial value</th>
<th>growth factor</th>
<th>range</th>
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</thead>
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4. Make a sketch of the graph for each exponential function in exercise 3.