Problems

16. Write out the partial sum $S_k$ for each series. Do not simplify.
   (a) $\sum_{n=0}^{\infty} (-1/2)^n$  
   (b) $\sum_{k=0}^{\infty} 2^k$  
   (c) $\sum_{i=1}^{\infty} (3 - 5i)$  
   (d) $\sum_{j=1}^{\infty} (2j)!/j^2$  
   (e) $\sum_{n=1}^{\infty} 1/\sqrt{n}$

17. Identify the value of $p$ and determine whether the series converges for each of the $p$-series, if any, in problem 16. Similarly, determine the ratio $r$ and convergence/divergence of any geometric series.

18. A series starts $1 + 1/2 + \cdots$; that is, the first two terms are 1 and $1/2$. Find a formula for the $n$-th term $a_n$ if the series is
   (a) an arithmetic series $\sum_{n=0}^{\infty} (kn + a_0)$ (i.e., find $k$ and $a_0$; $a_n = kn + a_0$).
   (b) a geometric series $\sum_{n=0}^{\infty} r^n$ (i.e., find $r$; $a_n = r^n$).
   (c) a $p$-series $\sum_{n=1}^{\infty} 1/n^p$ (i.e., find $p$; $a_n = 1/n^p$).

19. Find a formula for the N-th partial sum of $\sum_{n=0}^{\infty} (3/2)^n$. Does the series converge or diverges? Explain.

20. Determine whether the series converges or diverges. Evaluate, if the series converges.
   (a) $\sum_{n=0}^{\infty} -\frac{1}{n^2}$
   (b) $\sum_{n=0}^{\infty} (-3/5)^n$
   (c) $\sum_{n=1}^{\infty} \frac{3}{n^2}$
   (d) $\sum_{n=0}^{\infty} (1/7)^{-n}$

21. Determine whether the series converges or diverges. $e$ is the Euler number. $e \approx 2.7$.
   (a) $\sum_{n=1}^{\infty} \frac{1}{n^2}$
   (b) $\sum_{n=1}^{\infty} n^2$
   (c) $\sum_{n=1}^{\infty} n^{1/e}$
   (d) $\sum_{n=1}^{\infty} n^{-1/e}$

22. Determine whether the series converges or diverges. Evaluate, if the series converges.
   (a) $\sum_{n=1}^{\infty} \left( \frac{2}{n^2} - \frac{2}{(n+1)^2} \right)$
   (b) $\sum_{n=1}^{\infty} \frac{3 - n}{n^2}$
   (c) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$
   (d) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n+1}$

23. Find a geometric series that converges to 3/4. Is there a geometric series that converges to 1/3? If there is, give it. If there isn’t, determine all the real numbers that are given by a geometric series.

24. Find a formula for the N-th partial sum of the telescoping series $\sum_{n=0}^{\infty} [n!2^n - (n + 1)!2^{n+1}]$, and use the formula and a calculator to find the partial sums $S_0$, $S_1$, $S_2$, $S_3$. What appears to be the value of the series?

25. Find a formula for the coefficients $a_n$ in the power series representation $\sum_{n=0}^{\infty} a_n x^n$ for $e^{-x}$. (Recall that $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$.)

Answers to Selected Problems

17. (c) is a $p$-series with $p = 1/2$, hence it diverges. Geometric series are (a) $r = -1/2$, convergent, and (b) $r = 2$, divergent.
18. (a) $\sum_{n=0}^{\infty} ((-3/4)n + 1)$  
   (b) $\sum_{n=0}^{\infty} (1/4)^n$  
   (c) $\sum_{n=1}^{\infty} 1/n^2$

19. $S_n = (1 - (3/2)^{n+1})/(1 - 3/2)$ diverges since $(3/2)^{n+1} \to \infty$.
20. (a) Converges to $-15/4$. (b) Converges to 5/8. (c) Converges to $-15/4 - 3 = -3/4$ (d) Diverges.
21. (a) is the only convergent one of the four.
22. (a) Converges to 2. (b) Diverges. (c) Converges to $1 + 1/2$ (d) Converges to $\ln 2 - 1/2$.
23. The Geometric series with ratio $r = -1/3$ converges to 3/4. The possible sums of the Geometric series are the numbers in the interval $(1/3, \infty)$.
24. $S_N = 1 - (N + 1)!2^{N+1}$, $S_0 = -1$, $S_1 = -7$, $S_2 = -47$, $S_3 = -383$, and $S_N \to -\infty$.
25. $e^{-x} = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$.