CONSTRUCTIONS AND CLASSIFICATIONS
Part 4: Periodicity

If \( f(x) = f(x + p) \) for some constant \( p \) and all \( x \) in the domain, the function \( f(x) \) is said to be periodic with period \( p \). One way to describe periodic functions by their graphs is by “cutting and pasting”: “cut” any vertical strip of width \( p \), and the entire graph is reproduced by “pasting” copies of the “cut” together side by side. In other words, the graph of a periodic function repeats itself; mathematically, \( f(x) = f(x + np) \) for all integers \( n \).

1. Below is the graph of a periodic function. What is the smallest period? Explain.

2. (a) Give an example of a line in the Cartesian plane that is the graph of a periodic function. 

   (b) What is the smallest period? Explain.

   (c) Which lines represent periodic functions? Explain.
3. (a) Graph \( f(x) = x^2 \) below. (b) Is \( f(x) = x^2 \) periodic? Explain.

4. If we are careful, we can construct a periodic function of period \( p \) from any function by “cutting” a vertical strip of the graph of width \( p \) and “pasting” copies of the strip side by side.

(a) Try this with \( f(x) = x^2 \), using the strip \( 1/2 < x \leq 1 \).

(b) Try this with \( f(x) = x^2 \), using the strip \( 1/2 \leq x < 1 \).

(c) Explain why at least one of the endpoints of the interval for \( x \) must be omitted.